V-I Characteristic Formulas of Voltage Limiting Type SPD and Their Applications

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***Abstract*—The V-I characteristic formulas which are applicable to the SPD with ZnO varistor as its non-linear element are presented. The formulas describe the relationship between residual voltage and surge current of an SPD, and have distinct advantages as compared with previous formulas that their deviations from correctly measured values of voltage and current are generally not more than 3% over a current range of more than two decades. The methods and steps for determining the formulas via testing and calculations are also presented in this paper, and at last an example is given to show how to resolve related problems by use of the V-I characteristic formulas.**

I. Introduction

In terms of the related specifications of SPD, an SPD is a device that contains at least one nonlinear component that is intended to limit surge voltages and divert surge currents. Therefore the relation of residual voltage versus surge current is a most important application characteristic over a specified current range, which is, typically from $0.05I_n$ to 2.0 I_n (I_n refers to the nominal discharge current of SPD).

Generally the limiting voltage of an SPD (U_{SPD}) at a given surge current (I_P) is a sum of the voltage across the nonlinear component (U_{VR}) –herein is varistor(s) and a structural voltage drop (U_x):

$$U_{SPD}= U_{VR} + U_x \quad (1)$$

The U_x includes all voltage drops along the current path inside the SPD but for the voltage drop on the, varistor(s), the main inner parts are a thermal link, two connection terminals, and conductors between the varistor(s) and terminals. As for a typical module type SPD, U_x corresponds to a voltage drop of a resistor $R_x < 1\text{m}\Omega$which is about (1～2)% of the U_{SPD}. Therefore the V-I characteristic formula of the varistor(s) used in SPD can be approximately considered as the formula of the SPD ($U_{SPD}\approx U_{VR}$).

If more accuracy is needed, then the structural voltage drop has to be added ($U_{SPD}= U_{VR} + U_x$). Because testing for U_x is easy to do, therefore the next discussions of this paper will focus on the V-I characteristic formulas of the varistors used in the SPD.

As for most applications of the SPDs, their V-I characteristic formula should satisfy the three-requirements as below:1) It covers a peak current range not less than two decades 2) The deviations of the formula values from the correctly measured values of voltage and current of SPD or varistor samples are not more than 3%,or at most 5%. 3) The test and calculation steps for determination of the formulas should be easy to do.

It may be helpful to review briefly the history of varistor's V-I characteristic formulas. Due to the conduction mechanisms of a varistor being quite complex, up to now there is no a theoretical V-I formula which is appropriate to ZnO varistor.

At the discovery of ZnO varistors in 1968, only an empirical equation (2) was given[1]

$$I = \left(\frac{V}{C}\right)^{\alpha} \quad (2)$$

Where: V is the voltage on the varistor as a current I passing through it; C and α are constants which depend on individual varistor, and theαwas termed as "non-linear voltage exponent (index)" that is greater than "1.0". The equation (2) was rewritten as equation (3) or equation (4) in the IEC spec. [2].

$$U = CI^{\beta} \quad (3)$$

$$I = AU^{\chi} \quad (4)$$

Where: U is the voltage on the varistor as a current I passing thru it; C, β, A and γ(γ=1/β) are constants which depend on the individual varistor. It should be noted that theγin equation (4) is identical to the α in equation (2).

Unfortunately the above V-I equations serve no useful purpose for engineering calculations due to their applicable only to a very narrow current range in which theαvalue is considered as a constant, but in fact theαvalue of a varistor varies significantly with the current variation, figure 1 gave an example. It is seen from the curve of α_{2I} versus log I that at I≈200A (log I≈2.3) $\alpha_{2I}\approx 60$, while at I≈8000A (log I≈3.9) $\alpha_{2I}\approx 8$ (α_{2I} refers to an αvalue over a current range [I, 2I])

Some years later another V-I formula (5) was proposed [3,4] that was derived from an equivalent circuit for varistor, in terms of which a varistor may be considered as a non-linear resistor ($CI^{\beta-1}$) connected in series with a linear resistor (R_X), as showed by equation (5).

$$U = (CI^{\beta-1} + R_x \cdot)I = CI^{\beta} + R_x \cdot I \quad (5)$$

This formula do satisfied the requirement of "a current range not less than two decades", but its deviation was unsatisfied to engineering calculations.

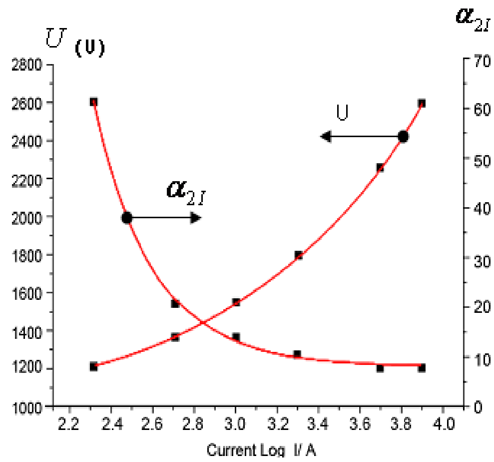


Fig. 1 Residual voltage (U) and voltage index α_{2I} vary with 8/20 current of (200～8000)A (Varistor－φ14mm,U_{1mA}=738V)

Some engineers including authors of this paper attempted to establish a V-I formula that satisfied the three-requirements as mentioned above by a mathematical fitting equation, but until author's paper [5] published, there was no such a fitting equation reported.

Apparently mathematical fitting is a common way and easy to do for a set of experimental data, but an appropriate formula was not obtained until our understanding of the varistor changed. In order to clarify this issue, an example was given as Table 1 and Fig.2.1, Fig.2.2, and Fig.2.3.

Table 1 Results of 8/20 surge current test

(Sample －Varistor,34×34mm,U_{1mA}=646.2V)

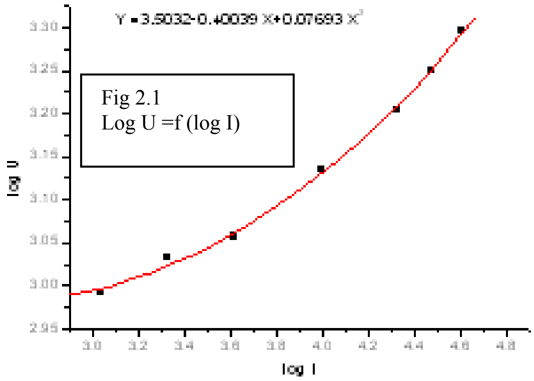
Measured data		Calculated data		
Current I (A)	Voltage U (v)	Log I	Log U	Log R R=U/I
1080	980	3.033	2.991	-0.0422
2120	1080	3.326	3.033	-0.2929
4120	1140	3.615	3.057	-0.5580
9920	1360	3.997	3.134	-0.8630
21200	1600	4.326	3.204	-1.222
30000	1780	4.477	3.250	-1.2267
40400	1980	4.606	3.297	-1.3097

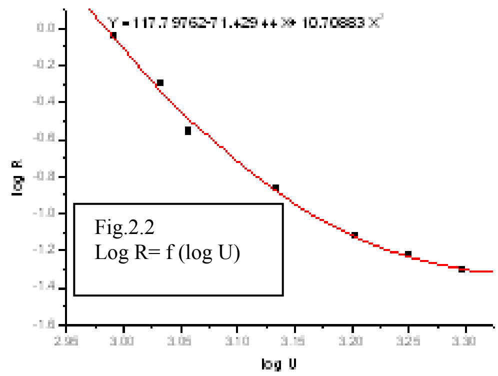
A set of data of (I～U) listed in Table 1 were measured peak values from a test on a sample of varistor. It is noted that the resistance (R=U/I) is a fictive resistance, because the U and I were not the same instant values. The above data were treated as polynomial fit to 2^{nd} order :

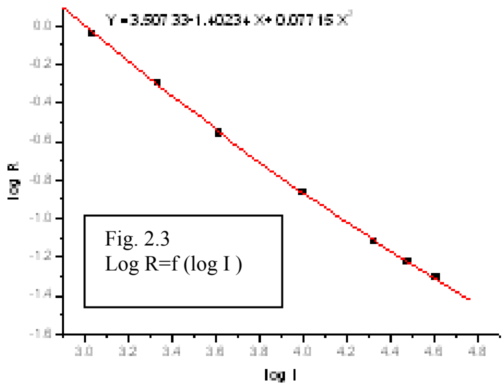
1) fit(log U) to (Log I), as illustrated by Fig.2.1.
2) fit (log R) to (Log U), as illustrated by Fig.2.2.
3) fit (log R) to (Log I), as illustrated by Fig.2.3.

It is clearly seen that the fitting curve Fig.2.3 is the best one because all test points were lying on the curve, moreover the curve was very close to a linear line. The following passage of this paper will show that the V-I characteristic formula derived from fitting (log R) to (Log I) can well satisfy the three-requirements , while the fitting equations obtained from fit(log U) to (Log I) or fit (log R) to (Log U) have a deviations of beyond 5%.

It is stressed that our idea of fitting (log R) to (Log I) came from our new understanding of varistor that "a varistor is more current dependent than voltage dependent", while Fig. 2.3 provided another evidence for this viewpoint.







II. FORMS OF THE V-I CHARACTERISTIC FORMULAS

The V-I characteristic formulas of a varistor that satisfy the three-requirements as above mentioned may be written as three different forms for different application purposes. They may be named "voltage form", "voltage ratio form", or "current form" .

1) V-I characteristic formula-voltage form:

$$U_{VR}=U_R\times I^B, \quad (6)$$

Where: U_{VR} = Residual voltage on a varistor as a surge current peak I of a specified waveform flows through it.

$$U_R=10^{A_0} \quad (7)$$

$$B=[(1+A_1)+A_2\times\log I]\times\log I \quad (8)$$

A_0, A_1 and A_2 are constants of a resistance equation (9)

$$\lg R=A_0+A_1\lg I+A_2(\lg I)^2 \quad (9)$$

2) V-I characteristic formula -residual voltage form:

$$k_{VR}=\frac{U_{VR}}{U_{1mA}} \quad (10)$$

Where: U_{1mA} = initial varistor voltage at d.c.1mA. The word "initial" refers to "as delivered" or "before degradation"

3) V-I characteristic formula – current form:

$$I=10^Y \quad (11)$$

$$y=\frac{-(1+A_1)\pm\sqrt{(1+A_1)^2+4A_2\cdot\log(U/U_R)}}{2A_2} \quad (12)$$

III. METHODS AND STEPS TO DETERMINE THE V-I CHARACTERISTIC FORMULAS

Manufacturers of varistors should provide the V-I characteristic formulas of delivered products for their users. Actually the characteristics of varistor units vary with type and production lot, even among the same lot, significant differences in the V-I characteristic may result from a 3%～5% difference in varistor voltage U1mA. Therefore, In order to obtain a V-I characteristic formula that agree well with the actual units, specimen should be properly sampled and subjected to specified tests followed by a proper math fitting for the tested data. An instance below will make the methods and steps easy to be understood.

There was a delivered varistor lot, the Part Number of which was 34S471 (34×34mm, U_{1mA}=470V), their V-I characteristic formulas at 8/20 surge current were requested.

1) Samples

Three samples were taken from the lot, whose varistor voltage U1mA lied in about the top, middle, and lowest value of the lot respectively, say 484.2V, 476.2V, 458.9V

2) Test

Perform tests on samples one by one for residual voltages at every specified peaks of the surge current .The actual readings of surge current peak (I) and residual voltage peak (U) should be recorded, as Table 2.

Table 2. Test record of current (I) and voltage (U)

Sample 458.9V		Sample 476.2V		Sample 494.2V	
I_L (A)	U_L (V)	I_M (A)	U_M (V)	I_H(A)	U_H (V)
300	620	296	660	288	680
608	660	600	680	592	720
904	680	896	700	880	740
1800	740	1800	760	1760	800
3760	800	3720	840	3680	860
7600	940	7360	960	7360	980
14080	1040	13970	1060	13760	1080
28400	1200	27600	1200	27800	1260
47200	1420	47200	1420	47200	1460

3) Calculations for A_0, A_1 and A_2 of each sample

Treat each set of data [I,U] of the three samples (Table 2) as following steps:

—Let $x=\log I$, and $y=\log R=\log(U/I)$

—Calculate fitting equation of y=f(x) in accordance with polynomial least-squares fitting by use of the software such as Origin, the polynomial function to second order, that resulted in equations like (13) or (14) for each of the three samples.

$$y=A_0+A_1x+A_2x^2=A_0+(A_1+A_2x)\cdot x \quad (13)$$

That is $\lg R=A_0+(A_1+A_2\lg I)\lg I$ (14)

The equation (14) is also named "Resistance formula".

The outcomes of the calculations for A_0, A_1 and A_2 were summarized in Table 3.

Table 3 Values of A_0, A_1 and A_2

Sample	458.9V	476.2V	494.2V
A0	2.965	2.982	3.014
A1	-1.1893	-1.1822	-1.1872
A2	0.04872	0.04652	0.04667

4) V-I characteristic formula of each sample

Substitute the values in Table 3 into the equations (6), (7) and (8), the V-I characteristic formula of each sample were obtained as equation (15.1), (15.2) and (15.3), additionally according to equation (10) the residual voltage ratio formulas were obtained as equation (16.1), (16.2) and(16.3)

Sample 458.9V : $U_{VR}=923\times I^B$ (15.1)

$$B = -0.1893 + 0.04872 \times \log I$$

$$k_{VR} = 2.011 \times I^{B} \qquad (16.1)$$

Sample 476.2V : $U_{VR} = 959 \times I^{B}$ (15.2)

$$B = -0.1822 + 0.04652 \times \log I$$

$$k_{VR} = 2.014 \times I^{B} \qquad (16.2)$$

Sample 494.2V : $U_{VR} = 1034 \times I^{B}$ (15.3)

$$B = -0.1872 + 0.04667 \times \log I$$

$$k_{VR} = 2.092 \times I^{B} \qquad (16.3)$$

5) Check deviations of the V-I characteristic formulas of each sample

Can the voltage formulas (15.1)～(15.3), or the voltage ratio formulas(16.1)～(16.3) satisfied the requirement of deviation? To answer this question let's make a comparison between the calculated voltage ratio KVRC and the measured voltage ratio KVRM , and define a percentage deviation as below :

$$D_{eV.}(\%) = \left(\frac{k_{VRC}}{k_{VRM}} - 1\right) \times 100\% \qquad (17)$$

Where: $k_{VRM} = \dfrac{measured \cdot voltage(see \cdot Table2)}{U_{1mA}}$

k_{VRC} =calculated voltage ratio by use of equation (16.1),(16.2) or (16.3) respectively at the currents that is the same as the currents for k_{VRM} (Table 2).

The percentage deviations of the three samples were summarized in Table 4, in which the KVRC and the KVRM for Sample 476.2V and 494.2V were omitted for simplicity reason.

Table 4 Derivations of the formulas (16.1), (16.2) and (16.3) from the actually measured values

458.9V			476.2V	494.2V
k_{VRM}	k_{VRC}	$D_{eV.}(\%)$	$D_{eV.}(\%)$	$D_{eV.}(\%)$
1.351	1.360	0.64	-0.88	0.89
1.438	1.426	-0.88	0.51	-0.72
1.482	1.478	-0.28	1.0	-0.31
1.613	1.600	-0.93	0.20	-1.05
1.743	1.776	1.86	0.02	1.36
2.048	2.007	-2.02	-2.13	-0.64
2.266	2.273	0.30	0.14	1.31
2.615	2.671	2.13	2.56	0.90
3.094	3.041	-1.74	-1.32	-1.22

All percentage deviations in Table 4 were less than 3% that had satisfied the requirement of deviation

5) Relations of residual voltage ratio to the U_{1mA} values of varistors and to the current level

In order to ascertain the differences in the voltage ratio cause by the different U1mA values, Table 5 was tabulated, in which the residual voltage ratios at each of specified current values of the three samples were listed, and the percentage deviations of (KVR-L) and (KVR-H) from KVR-N were also given.

Table 5, An example of KVR= f (U_{1mA}, I)

Current A	458.9V (L)		476.2V	494.2V (H)	
	KVR-L	%	KVR-N	%	KVR-H
200	1.336	-1.18	1.352	1.41	1.371
500	1.404	-0.84	1.416	0.99	1.430
1000	1.493	-0.47	1.50	0.67	1.510
2000	1.620	0	1.620	0.37	1.626
5000	1.861	0.70	1.848	0	1.848
10000	2.117	1.44	2.087	-0.24	2.082
20000	2.457	2.20	2.404	-0.50	2.392
40000	2.911	3.08	2.824	-0.78	2.802
50000	3.088	3.42	2.986	-0.83	2.961

Table 5 provided some more important information:

① As far as the discussed lot of the varistor is concerned, the formula (16.2) can also be used for the product of high-U_{1mA}, because of the deviations less than 2%, while it cannot be used for the product of low-U1mA because of the maximum deviation up to 3.42%. However if a deviation of 5% could be accepted, then the formula (16.2) can be used for all units of this lot.

② It is interesting to note that with the current varies from low level to high level, the residual voltage ratios of the low-U1mA varistors exhibit an upward variation (the deviations from minus to zero and to plus), while it is in an opposite way for the high-U1mA varistors. therefore at about the geometry medium current level (2kA～5kA ,in Table 5), the residual voltage ratios were almost independent of the U1mA values of the varistors.

IV. APPLICATIONS OF V-I CHARACTERISTIC FORMULAS

The V-I characteristic formulas discussed above can be a very useful tool for SPDs, ZnO arresters and varistors, especially for coordination calculations between SPDs. The following is an instance.

There was an SPD (1st SPD) in the power distributor, and a small sized SPD (2nd SPD) built in a tester that was powered by the distributor. Both SPDs were varistor- based type of the following properties:

1stSPD— 34×34mm, Imax=40kA,U1mA=620V

$U = 1702 \cdot I^B$,where $B = -0.247 + 0.05436 \times \log I$

2nd SPD—φ10mm, Imax=3.5kA,U1mA=620V

$U = 1091 \cdot I^B$, where $B = -0.1812 + 0.06662 \times \log I$

The problem is that as the1st SPD is impinged by an 8/20-20kA surge current, can the 2nd SPD be survival? To simplify the problem, the connection wires between the two SPDs were neglected, so that they are treated as a parallel combination. hence by use of the formulas (11) and (12), the currents passing through the two SPDs can be obtained at some arbitrary voltages from 950V to 1050V, see Table 6. Figure 3 gave a further description of the current share. From Table 6 and Figure 3, it is concluded that the 2nd SPD will be safe in case of an impinging current of 20kA into the 1st SPD, because the current thru 2nd one is 2488A, smaller than its I_{max} (3.5kA). A specially performed test has demonstrated that the current share listed in table 6 is true.

Table 6 Currents thru the two paralleled SPDs

Voltage	Current (A)			Ratio
(V)	1st [I1]	2nd [I2]	total(I1+I2)	I1/I2
950	958.9	216.1	1175	4.43
1050	2591.	422.9	3014	6.13
1150	4974	695.4	5669	7.15
1250	8201	1037	9238	7.91
1350	12344	1449	13792	8.52
1450	17461	1932	19393	9.04
1550	23600.	2488	26089	9.48
1650	30802.	3117	33919	9.88

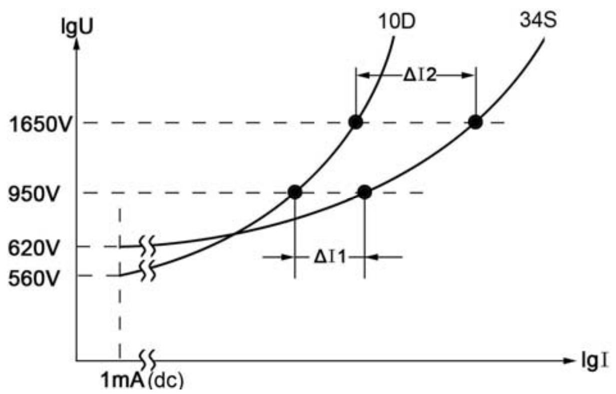


Figure 3 Current share between 1st and 2nd SPD

In addition, giving attetion to the following matters is also necessary:

1) Prior to performing the tests for V-I characteristic formula,varistors should be subjected to an aging process to stabilize their properties, particularly U1mA values.

2) During the service life, the U1mA value is going down gradually, while the residual voltage at a specified current is going up gradually with a smaller percentage variation than U_{1mA} variation . that means the V-I characteristic is changing accordingly.

3) As for a given unit of varistor, the constants of A_0, A_1 and A_2 in the V-I characteristic formula depend to some extent on the waveform of test current, therefore, an independent test should be carried out for a changed waveform.

V. CONCLUSIONS

Recent years some new insights have been gained into the non-linear resistance properties of ZnO varistors including new V-I characteristic formulas which described the relationship between the current and voltage of a varistor

It has been demonstrated that the V-I characteristic formulas have great significance for solving the problems in following aspects:

— Provide practical voltage limiting characteristics of varistors or SPDs to be used ,that is the basis of overvoltage protection design.

—realize full coordination between SPDs in a easy way.

—Guide the practices of combining varistors in parallel to raise their ratings of surge current or energy.

—By use of the information involved in the formulas, the fabrication processes of varistors may be managed and controlled effectively.

— The formulas can also be used for new product development. An application requested V-I characteristic is planned in advance followed by a serial developing works to realize it.

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